# Non-Representative Agents

Analyzing the Impact of Representative Agent Assumptions on DSGE Models

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#### Abstract

Much of modern macroeconomics uses models that rely on representative agent assumptions. This paper analyzes the effect of these assumptions on macroeconomic variables through a simple model that introduces a hand-to-mouth agent to the economy. We find that this Two Agent New Keynesian (TANK) model creates substantially different outcomes than a Representative Agent New Keynesian (RANK) model that are much more volatile. We also find that in the TANK, both fiscal and monetary policy have effects that aren't seen from the RANK.

## 1 Introduction

Much of modern macroeconomic theory is based on the interactions of different representative agents. Instead of the 336 million people in the US ("U.S. and World Population Clock" 2024), a model will have a single "representative household" interact with a "representative firm" in a "representative industry." This assumption has a number of justifications, from pragmatic one about model tractability to theoretic ones about aggregation (Hartley 1996), however it has meant that historically most macroeconomic theory has ignored both how wealth and income distributions arise and how unequal distributions affect an economy.

 $<sup>\</sup>label{eq:cond} $$ Replication files are available at https://github.com/GavinEngelstad/Spring2024MacroModeling. If you have questions, contact gengelst@maclester.edu.$ 

This paper will peel back these representative agent assumptions and explore how economic outcomes are affected by the addition of household heterogeneity to a model. It finds that representative agent models misrepresent certain economic outcomes, especially related to levels of volatility in the economy, and make misguided policy recommendations, especially in regard to fiscal policy.

## 2 Literature Review

#### 2.1 Representative Agent Macro

The first DSGE models used representative agents to simulate an economy where households owned capital, worked an equilibrium number of hours, and made consumption decisions (Kydland and Prescott 1982). When the model was altered to add unemployment, instead of adding a level of heterogeneity to the model with different employed and unemployed households, it was done using a labor lottery and probabilities (Hansen 1985; Rogerson 1988). These assumptions allowed for a tractable model, even considering the weaker computational power, but meant that unemployed and employed people were lumped together into a single agent in the model.

For the next two decades, research in macro was done by making modifications to this representative agent foundation. The addition of different frictions, market structures, and interactions between agents allowed economists to better analyze how the macroeconomy as a whole functioned, but still, for the most part, ignored differences in agents of the same type (Clarida, Gali, and Gertler 1999; Smets and Raf Wouters 2003; Christiano, Eichenbaum, and Evans 2005; Smets and Rafael Wouters 2007). This became the Representative Agent New Keynesian (RANK) model.

Representative agent modeling, however, has structural issues that cannot be addressed by just adding more, increasingly complex frictions. Early in the development of these models, it was known that the aggregation of heterogeneity to create a representative agent would always lead to miscalculation of economic effects and was an oversimplification that weakened overall models (Geweke 1985; Kirman 1992; Hands 2017). More recently, An, Chang, and Kim 2009 showed that identically calibrated representative and heterogeneous agent models lead to different economic outcomes. These issues together mean, at least in part, an overreliance on these models may have led economists to be unaware of the recession threat pre-great recession (Treeck and Sturn 2012).

#### 2.2 Hand to Mouth Households

The simplest heterogeneous agent models have two different types of agents. Campbell and Mankiw 1989 was the first of such models, and split the households evenly into two groups, one of which acted similar to a representative agent and the other acted in a "hand to mouth" manner, consuming everything they earned each period. It found that such a model was better able to match consumption patterns than representative agent models.

This type of model became the Two-Agent New Keynesian (TANK) model. TANK models have been used to explain why Ricardian equivalence can break and consumption increases with government spending (Galí, López-Salido, and Vallés 2007), monetary policy can have unexpected results (Bilbiie 2007; Walsh 2017), shocks can become amplified intertemporally (Bilbiie 2018), and inequality arises (Troch 2014; Broer et al. 2016). A three-agent model (THRANK) similarly finds that agent heterogeneity causes monetary policy shocks to be amplified and cause increasing inequality (Eskelinen 2021). Essentially, these sorts of models find that heterogeneous agent model foundations cause substantially different results than representative agent ones.

This paper will expand on this research and create a TANK that uses a Hand-to-Mouth household following Galí, López-Salido, and Vallés 2007. Using this type of model, we'll be able to analyze these same macroeconomic outcomes. We'll focus on the effects of Government spending and monetary policy shocks as well as aggregate volatility in the economy.

#### 2.3 Real Heterogeneity

Despite having more differentiation than RANK models, TANK (and THRANK) models still suffer from aggregating many individuals in the economy into just a handful of economic agents. The alternative to this is a Heterogeneous Agent New Keynesian (HANK). These models assume there is either a continuous distribution of agents (Preston and Roca 2007; McKay, Nakamura, and Steinsson 2015; McKay and Reis 2016b; Kaplan, Moll, and Violante 2018; Bilbiie 2019; Acharya and Dogra 2020; Aliprantis, Carroll, and Young 2022)<sup>1</sup> or a large finite number of them (Werning 2015; Auclert, Rognlie, and Straub 2023).

Households in HANK models tend to be more reactive, planing less for the future (McKay, Nakamura, and Steinsson 2015; McKay and Reis 2016b; Kaplan, Moll, and Violante 2018), and, after shocks, consume at levels that aren't easily approximated by RANK models (Preston and Roca 2007; Werning 2015; Acharya and Dogra 2020). These sorts of models are also able to replicate real-world wealth inequality, both for individuals (Auclert, Rognlie, and Straub 2023) and different subsets of the population (Aliprantis, Carroll, and Young 2022).

We do see, however, that HANK models can be easily approximated by TANK models and are exceptionally difficult to solve (Reiter 2009). Bilbiie 2018, using the HANK model outlined in Bilbiie 2019, found that a TANK can approximate the monetary policy effects and inequality in a simplistic HANK model. Debortoli and Galí 2018 found that TANKs can generally approximate the aggregate outcomes of a HANK.

Therefore, in our analysis we'll focus on comparisons between a RANK and TANK model. This doesn't include a complete layer of heterogeneity, but should be able to approximate the predictions of a HANK and be much simpler to solve.

## 3 The Models

This paper will present two models, a RANK and a TANK, that are set up identically except for the level of household heterogeneity. The RANK model uses a standard New-Keynesian setup. The TANK model adds a hand-to-mouth representative household to the RANK. For the sake of brevity, only the model setup and characterization are presented in the paper. For how the model was solved, see Appendix A.

<sup>1.</sup> Which is then (typically, some papers like Bilbiie 2019 find an analytical solution) estimated using a large, finite number of agents (Algan et al. 2014)

#### 3.1 Firms, the Government, and the Central Bank

Since the firm, government, and central bank problems are the same in both models, they'll be presented together.

**Final Goods Firm.** The model has a single representative, perfectly competitive final goods firm. The final goods firm combines intermediate goods according to the production function

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}} \mathrm{d}j\right)^{\frac{\psi}{\psi-1}}$$

where  $Y_t$  is aggregate output and  $y_{j,t}$  is output for the *j*th intermediate good.

The profit maximization problem gets that

$$y_{j,t} = Y_t \left(\frac{P_t}{p_{j,t}}\right)^{\psi}$$
$$P_t = \left(\int_0^1 p_{j,t}^{1-\psi}\right)^{\frac{1}{1-\psi}}$$

where  $P_t$  is the aggregate price level of the model economy and  $p_{j,t}$  is the price of the *j*th intermediate good.

Intermediate Goods Firms. The model is populated by a unit continuum of monopolistically competitive intermediate goods firms  $j \in [0, 1]$ . Each intermediate goods firm produces their variety according to the production function

$$y_{j,t} = Z_t n_{j,t}$$

where  $n_{j,t}$  is the labor used to produce the *j*th intermediate good and  $Z_t$  is the economy's efficiency and follows the stochastic process

$$\log Z_t = \rho_Z \log Z_t + \varepsilon_{Z,t}$$

where  $\varepsilon_{Z,t} \sim \mathcal{N}(0, \sigma_Z)$ .

The cost minimization problem gets that

$$W_t = Z_t \Lambda_t$$

where  $W_t$  is the real wage and  $\Lambda_t$  is the real marginal cost of production across all intermediate goods.

Each period, intermediate goods firms follow a Calvo rule and update prices with probability  $1 - \theta$  (Calvo 1983). When updating their price, firms pick  $P_t^*$  to maximize expected profits from then until they are allowed to update their price again. Therefore,

$$\frac{P_t^*}{P_t} = \frac{\psi \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \Lambda_s Y_s \left(\frac{P_s}{P_t}\right)^{\psi}}{(1-\psi) \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} Y_s \left(\frac{P_s}{P_t}\right)^{\psi-1}}.$$

where  $R_{t,s}$  is the real net return on bonds between periods t and s.

Since these firms are monopolistically competitive, they can make a profit. Each firm j uses this profit to finance dividends  $d_{j,t}$  to households such that

$$d_{j,t} = y_{j,t}p_{j,t} - P_t W_t n_{j,t}.$$

**Firms Aggregated.** Using the intermediate goods firm pricing rule and the final goods firm price aggregator, we get

$$1 = \theta \pi_t^{\psi - 1} + (1 - \theta) \left(\frac{P_t^*}{P_t}\right)^{1 - \psi}$$

where  $\pi_t$  is inflation and equal to

$$\pi_t = \frac{P_t}{P_{t-1}}.$$

Additionally, defining aggregate labor usage  $N_t$  as

$$N_t = \int_0^1 n_{j,t}$$

means the intermediate goods firm production function becomes

$$Y_t = Z_t N_t$$

Finally, defining the aggregate real dividends  $D_t$  as

$$D_t = \frac{\int_0^1 d_{j,t} \mathrm{d}j}{P_t}$$

and integrating the expression for dividends gets

$$D_t = Y_t - W_t N_t.$$

**Government.** The role of the government is to hold some amount of debt,  $B_t$ , spend according to the rule

$$G_t = Y_t g_t$$

where  $g_t$  is an exogenous variable that evolves according to

$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\overline{g} + \varepsilon_{g,t}$$

where  $\varepsilon_{g,t} \sim \mathcal{N}(0, \sigma_g)$ , and impose a lump sum tax on households to finance interest on debt and spending such that

$$T_t + B_t = R_{t-1}B_{t-1} + G_t.$$

Central Bank. The Central Bank sets the interest rate according to the simple Taylor rule

$$I_t = \overline{R}\pi_t^\omega \xi_t$$

where  $\overline{R}$  is the target real interest rate,  $\xi_t$  is a stochastic monetary policy shock that follows

$$\log \xi_t = \rho_{\xi} \log \xi_{t-1} + \varepsilon_{\xi,t},$$

and  $I_t$  is the net nominal interest rate such that

$$R_t = \mathbb{E} \frac{I_t}{\pi_{t+1}}.$$

This follows McKay and Reis 2016b and excludes the normal output gap coefficient to simplify the model.<sup>2</sup>

#### 3.2 RANK Model

The RANK follows a standard New-Keynesian setup with a lifetime-optimizing representative household.

Representative Household. The Representative Household has preferences given by

$$\sum_{t=1}^{\infty} \beta^t U(C_t, L_t)$$

where  $C_t$  is consumption and  $L_t$  is the amount of labor supplied by the household. The utility function is given by

$$U(C_t, L_t) = \frac{C_t^{1-\eta}}{1-\eta} - \phi \frac{L_t^{1+\chi}}{1-\chi}.$$

Each period, the household has the budget constraint

$$C_t + B_t = R_{t-1}B_{t-1} + W_t L_t + D_t - T_t$$

where  $B_t$  is the real bond holdings of the household.

Solving the household maximization problem gets the Euler Equation

$$1 = \beta R_t \mathbb{E} \frac{C_t^{\eta}}{C_{t+1}^{\eta}}$$

and intratemporal condition

$$\phi L_t^{\chi} C_t^{\eta} = W_t.$$

<sup>2.</sup> Many other New-Keynsian papers zero out the exponent to that coefficient to similar effect (Sims and Wu 2019). This just makes it more explicit.

Market Clearing. Market clearing requires the goods market to clear

$$Y_t = C_t + G_t$$

and the labor market to clear

$$L_t = N_t.$$

By Walrus's law, we'll ignore the bond market.

**Competitive Equilibrium.** From these, a competitive equilibrium of the model is a set of wages  $\{W_t\}_{t=0}^{\infty}$ , household allocations  $\{C_t, L_t, B_t\}_{t=0}^{\infty}$  that satisfy the household conditions

$$C_t + B_t = R_{t-1}B_{t-1} + W_tL_t + D_t - T_t$$
$$W_t = \phi L_t^{\chi} C_t^{\eta}$$
$$1 = \beta R_t \mathbb{E} \frac{C_t^{\eta}}{C_{t+1}^{\eta}},$$

firm allocations  $\{Y_t, N_t, \Lambda_t, D_t, \pi_t, \frac{P_t^*}{P_t}\}_{t=0}^{\infty}$  that satisfy the conditions for the final and intermediate goods firms

$$\begin{split} \Lambda_t &= Z_t W_t \\ 1 &= \theta \pi_t^{\psi - 1} + (1 - \theta) \left(\frac{P_t^*}{P_t}\right)^{1 - \psi} \\ \frac{P_t^*}{P_t} &= \frac{\psi \mathbb{E} \sum_{s=t}^\infty \theta^{s-t} R_{t,s}^{-1} \Lambda_s Y_s \left(\frac{P_s}{P_t}\right)^\psi}{(1 - \psi) \mathbb{E} \sum_{s=t}^\infty \theta^{s-t} R_{t,s}^{-1} Y_s \left(\frac{P_s}{P_t}\right)^{\psi - 1}} \\ Y_t &= Z_t N_t \\ D_t &= Y_t - W_t N_t, \end{split}$$

government allocations  $\{\tau_t, G_t\}_{t=0}^{\infty}$  that satisfy the government conditions

$$G_t + R_{t-1}B_{t-1} = T_t + B_t$$
$$G_t = Y_t g_t,$$

and Central Bank allocations  $\{I_t, R_t\}_{t=0}^{\infty}$  that satisfy the Taylor Rule and real interest rate definition

$$I_t = \overline{R} \pi_t^{\omega} \xi_t$$
$$R_t = \mathbb{E} \frac{I_t}{\pi_{t+1}}$$

such that the goods and labor markets clear

$$Y_t = C_t + G_t$$
$$L_t = N_t$$

subject to the exogenous processes

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t}$$
$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_{\xi,t}$$
$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\overline{g} + \varepsilon_{g,t}.$$

To solve the system, we use Linear Time Iteration from Rendahl 2017 to find a policy function around the zero inflation steady state. For more information, see Appendix B.1.

#### 3.3 TANK Model

The TANK is identical to the RANK except for the addition of "Hand-to-Mouth" (HtM) households. These households don't optimize intertemporally and instead just consume everything they have each period.

Hand-to-Mouth Households. HtM households make up a fraction  $\mu$  of the population and

don't have access to the bond market. Therefore, each period they maximize utility given by

$$U = \frac{C_t^{H1-\eta}}{1-\eta} - \phi \frac{L_t^{H1+\chi}}{1+\chi}$$

where  $C_t^H$  is consumption by HtM households and  $L_t^H$  is labor supplied by HtM households subject to the budget constraint

$$C_t^H = W_t L_t^H + D_t - T_t.$$

Therefore, their intratemporal constraint is

$$W_t = \phi L_t^{H\chi} C_t^{H\eta}.$$

Saver Households. "Saver" households make up  $1 - \mu$  of the population and have access to the bond market to optimize intertemporally. They have preferences given by

$$\sum_{t=1}^{\infty} \beta^t U(C_t^S, L_t^S)$$

where  $C_t^S$  is consumption of savers,  $L_t^S$  is the amount of labor supplied by savers, and

$$U(C_t, L_t) = \frac{C_t^{S1-\eta}}{1-\eta} - \phi \frac{L_t^{S1+\chi}}{1-\chi}.$$

Each period, savers have the budget constraint

$$C_t^S + B_t^S = R_{t-1}B_{t-1}^S + W_t L_t^S + D_t - T_t$$

where  $B_t^S$  is the real bond holdings of the saver.

The household maximization problem gets the Euler Equation

$$1 = \beta R_t \mathbb{E} \frac{C_t^{S\eta}}{C_{t+1}^{S\eta}}$$

and intratemporal condition

$$(1-\tau_t)W_t = \phi L_t^{S\chi} C_t^{S\eta}.$$

Households Aggregated. Aggregate consumption is given by

$$C_t = \mu C_t^H + (1 - \mu) C_t^S.$$

Aggregate labor supply is given by

$$L_t = \mu L_t^H + (1 - \mu) L_t^S.$$

Finally, aggregate bonds are given by

$$B_t = (1 - \mu)B_t^S.$$

Market Clearing. Market clearing requires the goods market to clear

$$Y_t = C_t + G_t,$$

and the labor market to clear

$$N_t = L_t.$$

By Walrus's law, we'll ignore the bond market.

**Competitive Equilibrium.** A competitive equilibrium is a set of real wages  $\{W_t\}_{t=0}^{\infty}$ , HtM household allocations  $\{C_t^H, L_t^H\}_{t=0}^{\infty}$  that satisfy the HtM household conditions

$$C_t^H = W_t L_t^H + D_t - T_t$$
$$W_t = \phi L_t^{H\chi} C_t^{H\eta},$$

saver household allocations  $\{C_t^S, L_t^S, B_t^S\}_{t=0}^{\infty}$  that satisfy the saver household conditions

$$C_t^S + B_t^S = R_{t-1}B_{t-1}^S + W_t L_t^S + D_t - T_t$$
$$W_t = \phi L_t^{S\chi} C_t^{S\eta}$$
$$1 = \beta R_t \mathbb{E} \frac{C_t^{S\eta}}{C_{t+1}^{S\eta}},$$

aggregate household indicators  $\{C_t,L_t,B_t\}_{t=0}^\infty$  that satisfy the definitions

$$C_{t} = \mu C_{t}^{H} + (1 - \mu) C_{t}^{S}$$
$$L_{t} = \mu L_{t}^{H} + (1 - \mu) L_{t}^{S}$$
$$B_{t} = (1 - \mu) B_{t}^{S},$$

firm allocations  $\{Y_t, N_t, \Lambda_t, D_t, \pi_t, \frac{P_t^*}{P_t}\}_{t=0}^{\infty}$  that satisfy the conditions for the final and intermediate goods firms

$$\begin{split} \Lambda_t &= Z_t W_t \\ 1 &= \theta \pi_t^{\psi - 1} + (1 - \theta) \left(\frac{P_t^*}{P_t}\right)^{1 - \psi} \\ \frac{P_t^*}{P_t} &= \frac{\psi \mathbb{E} \sum_{s=t}^{\infty} \theta^{s - t} R_{t,s}^{-1} \Lambda_s Y_s \left(\frac{P_s}{P_t}\right)^{\psi}}{(1 - \psi) \mathbb{E} \sum_{s=t}^{\infty} \theta^{s - t} R_{t,s}^{-1} Y_s \left(\frac{P_s}{P_t}\right)^{\psi - 1}} \\ Y_t &= Z_t N_t \\ D_t &= Y_t - W_t N_t, \end{split}$$

government allocations  $\{T_t,G_t\}_{t=0}^\infty$  that satisfy the conditions for the government

$$G_t + R_{t-1}B_{t-1} = T_t + B_t$$
$$G_t = g_t Y_t,$$

and Central Bank allocations  $\{I_t, R_t\}_{t=0}^{\infty}$  that satisfy the Taylor Rule and real interest rate definition

$$I_t = R\pi_t^{\omega}\xi_t$$
$$R_t = \mathbb{E}\frac{I_t}{\pi_{t+1}}$$

such that the goods and labor markets clear

$$Y_t = C_t + G_t$$
$$N_t = L_t$$

subject to the exogenous processes

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t}$$
$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_{\xi,t}$$
$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\overline{g} + \varepsilon_{g,t}.$$

To solve the system, we use Linear Time Iteration from Rendahl 2017 to find a policy function around the zero inflation steady state. For more information, see Appendix B.2.

## 4 Calibration

The main goal with our calibration in this paper is to have a consistent framework to make comparisons between the two models. Therefore, to calibrate the model, we'll pick values to be consistent with other literature instead of trying to match the real world as closely as possible. Our calibrated values are in Table 4.1. These parameter values are chosen based on McKay and Reis 2016b, McKay 2018, and Galí, López-Salido, and Vallés 2007.

Parameter	Value	Description	RANK	TANK
$\beta$	0.995	Household Intertemporal Discounting	$\checkmark$	$\checkmark$
$\eta$	1	Relative Risk Aversion Parameter	$\checkmark$	$\checkmark$
$\phi$	2	Importance of Leisure Relative to Consumption	$\checkmark$	$\checkmark$
$\chi$	2	Inverse Frisch Elasticity of Labor	$\checkmark$	$\checkmark$
$\psi$	6	Elasticity of Substitution Between Varieties	$\checkmark$	$\checkmark$
heta	0.75	Calvo Rule Parameter	$\checkmark$	$\checkmark$
ω	1.5	Taylor Rule Inflation	$\checkmark$	$\checkmark$
$\overline{R}$	1.005	Goal Real Interest Rate	$\checkmark$	$\checkmark$
$ ho_Z$	0.95	$Z_t$ Regression to Steady State	$\checkmark$	$\checkmark$
$\sigma_Z$	0.01	Standard Deviation in $\varepsilon_{Z,t}$	$\checkmark$	$\checkmark$
$ ho_{\xi}$	0.8	$\xi_t$ Regression to Steady State	$\checkmark$	$\checkmark$
$\sigma_{\xi}$	0.01	Standard Deviation in $\varepsilon_{\xi,t}$	$\checkmark$	$\checkmark$
$\overline{g}$	0.25	Steady State Government Fraction of Output	$\checkmark$	$\checkmark$
$ ho_q$	0.9	$g_t$ Regression to Steady State	$\checkmark$	$\checkmark$
$\sigma_{g}$	0.01	Standard Deviation in $\varepsilon_{g,t}$	$\checkmark$	$\checkmark$
$\ddot{\mu}$	0.5	Fraction of HtM Households		$\checkmark$
В	0	Steady State Bonds		$\checkmark$

Table 4.1: Parameter Values in the Models

## 5 Results

#### 5.1 Government Spending Shock

The consumption responses to a government spending shock are shown in Figure 5.1. In the RANK, we see a slight drop in consumption that slowly converges to the steady state value, consistent with what would be expected by Ricardian Equivalence. In the TANK, we see a sharp increase in consumption directly after the shock, but this increase isn't sustained, and consumption very quickly converges to the RANK value.

This change is entirely driven by the change in consumption by HtM households. After the shock, saver households follow Ricardian Equivalence and never consume more than steady state consumption in response to the shock. HtM households, however, experience a sudden, sharp increase in consumption.

This makes sense, since Ricardian Equivalence is driven by households optimizing based on expectations about taxes in the future. In this case, since HtM households don't optimize or make decisions intertemporally, it holds that Ricardian equivalence should break for HtM households and



Figure 5.1: Consumption Response to Government Spending Shock

overall when there is a large enough fraction of the population that's HtM.

To test this idea, we also run the model with  $\mu$  set to 0.25 and 0.75. In this way, we get to test whether there is a critical mass of HtM households for which we observe this break in Ricardian Equivalence.

The results to this are shown in Figure 5.2. We see that with all values for  $\mu > 0$ , we observe a similar increase in consumption, but with a lower  $\mu$ , Ricardian equivalence still holds and consumption never goes above the steady state value. Specifically, with  $\mu = 0.25$ , the period 2 level of consumption is about 0.12% below steady state. This is higher than in the RANK, which is about 0.17% below the steady state, but still means that the increase in government spending never caused a corresponding increase in consumption.



Figure 5.2: Consumption Response to Government Spending Shock by  $\mu$ 

Interestingly, we see limited consumption difference between period 2 consumption when  $\mu = 0.5$ and  $\mu = 0.75$ . We do, however, see Ricardian equivalence break in the first period as well with the higher  $\mu$  and take longer to converge to the consumption path for the RANK. This suggests that a higher mass of HtM households will significantly affect when the consumption increase will happen, but will only marginally affect the maximum magnitude of the increase.

Therefore, consistent with Galí, López-Salido, and Vallés 2007, we find that with a sufficiently high mass of HtM households, Ricardian equivalence can break and that suggest fiscal policy can be an effective tool to spur increases in consumption, albeit temporarily.

#### 5.2 Interest Rate Shock

In response to a monetary policy shock, the TANK has a much stronger disinflation response than the RANK (Figure 5.3). This, through our Taylor Rule, causes the interest rate to rapidly decrease and then increase prior to stabilizing towards the steady state. The RANK, in contrast, has a relatively smooth response to the shock, with an uptick in the interest rate immediately at the start before converging to the steady state.

This large Taylor Rule response to an interest rate shock explains the weird behavior of the TANK in response to an interest rate shock. In contrast to the RANK, which rapidly changes at the period of the shock then slowly converges towards the steady state, both inflation and output in the TANk have significant, sudden declines to values well below the RANK, then approach the



Figure 5.3: Response to Interest Rate Shock

steady state much more quickly.

This is consistent with the idea of an "inverted Taylor Principle" from Bilbiie 2007, which finds that, depending on parameterization, a TANK can have abnormal behavior in response to a monetary policy shock through this inflation response mechanism causing the interest rate response to flip. Therefore, we find that TANKs can have weirder, less predictable behavior than RANKs in response to interest rate shocks.

#### 5.3 TFP Shock

The response to a consumption shock on output and agent-level consumption is shown in Figure 5.4.

After the shock, we observe a much sharper immediate increase in consumption in the TANK compared to the RANK, likely due to the fact that the HtM portion of the households in the TANK aren't optimizing intertemporally and therefore aren't consumption smoothing. The RANK, much



Figure 5.4: Output Response to TFP Shock

like with the other variables after other shocks, has a gradual, sustained decrease in output that converges back to the steady state.

Like we saw with the government spending shock, much of this change is driven by changes in consumption patterns for HtM households. In contrast, savers retain a similar post-shock consumption pattern to the representative household in the RANK,<sup>3</sup> with some slight kinks at the start, probability caused by the macroeconomic effects of the dramatic changes in behavior by the HtM households.

Curiously, the labor effect is driven by the opposite agent. Figure 5.5 shows the labor impacts of the shock. Like would be expected with log preferences,<sup>4</sup> labor supply in the RANK remains constant after the TFP shock. In the TANK, however, we observe the labor supply increasing, then decreasing, then converging to the steady state.

Looking at the agent-level breakdown of the shock, we observe that saver agents drive most of this, and HtM agents have a comparatively small change in labor supply.<sup>5</sup> This is likely because saver households make their labor decisions based on both what they want to consume today and their expected consumption in the next period, so their level of savings, expected inflation, and the interest rate all play a role in this decision, not just the wage like for the HtM household. This multitude of potentially volatile factors that play into the decision for how much savers work mean that the decision becomes much more volatile.

Therefore, agent-level behavior causes aggregate variables in a TANK are much more volatile in response to a TFP shock than in a RANK.

#### 5.4 Simulation

To analyze the difference in aggregate economic outcomes in the RANK versus the TANK, we'll run a 1,000 period simulation of both economies with the same paths for the stochastic variables (Figure D.1). In this way, we see the differences in economic outcomes for the RANK and TANK subject to the same shocks.

<sup>3.</sup> Which, since  $g_t$  isn't affected by the shock is proportional to RANK Y.

<sup>4.</sup> With  $\eta = -1$ , the FOCs for the utility function are the same as would be expected with log preferences, meaning that's essentially what we're solving here.

<sup>5.</sup> This effect could be seen in  $L_t^{\bar{H}}$  vs  $L_t^S$  graphs for the other shocks too; it's not unique to a TFP shock.

The first 100 periods of this simulation are shown in Figure 5.6. Across the board, we see the RANK and TANK follow approximately the same paths, but the TANK is much more volatile than the RANK.

Looking at the agent breakdown, we see this volatility in consumption is primarily driven by HtM households, but the volatility in labor supply instead comes from saver households. This suggests that savers exhibit consumption smoothing as expected but, likely because of a combination of volatility in wages, inflation, and the real interest rate, all of which are substantial in the TANK, change how much labor they're willing to supply quite often throughout the simulation.

The statistics in Table 5.1 show this same idea. Across all variables, the means of the macroeconomic variables are about the same between the two models, but the standard deviations in the



(a) TANK vs RANK

Figure 5.5: Labor Response to TFP Shock



Figure 5.6: Aggregate Economic Variables from First 100 Periods of 1,000 Period Simulation

TANK are substantially larger in the TANK than the RANK. We also see that consumption for HtM agents is more volatile with a standard deviation almost 2.5 times that of savers, but labor

	$\mathbf{R}_{\mathbf{A}}$	ANK	TANK		
	Mean	St. Dev.	Mean	St. Dev.	
$\overline{Y_t}$	0.821	0.030	0.821	0.052	
$N_t$	0.822	0.014	0.822	0.046	
$-L_t^H$			0.822	0.028	
$-L_t^S$			0.822	0.068	
$C_t$	0.617	0.022	0.617	0.039	
$-C_t^H$			0.617	0.071	
$-C_t^S$			0.617	0.029	
$G_t$	0.204	0.010	0.204	0.015	
$\pi_t$	1.000	0.020	1.000	0.036	
$W_t$	0.833	0.047	0.834	0.140	
$I_t$	1.005	0.014	1.005	0.038	
$R_t$	1.005	0.004	1.005	0.017	

Table 5.1: Simulation Summary Statistics

supply is more volatile for savers.

These simulation results are substantially more volatile than the real world (Kydland and Prescott 1982), meaning better calibration or parameter estimation is needed for this TANK to be a viable model for analyzing the real world, but does suggest that RANK models will misestimate aggregate economic outcomes.

## 6 Conclusion

Representative agent assumptions are ubiquitous in macroeconomic modeling since they make model construction and solution much easier, but are flawed. In this paper, we found that even the simplest addition of heterogeneity will cause alternate economic outcomes after policy intervention and more volatility in the economy.

Especially important is this papers ideas relating to the effectiveness of fiscal policy at increasing consumption and breaking Ricardian equivalence. It's known that HtM-style households do exist in the real world (Aguiar, Bils, and Boar 2023), so the fact that the addition of this style of agents to the model makes fiscal policy more effective means governments and policymakers have an additional tool to affect the economy in potentially desirable ways. This paper also finds that monetary policy may have unexpected effects within a TANK than a RANK, suggesting central banks should use more complex models with more layers of heterogeneity when they set interest rates to avoid unexpected results.

#### 6.1 Limitations

This paper has a number of limitations that affect its analysis.

First, the primary goal of the model calibration was to make sure the two models were set up uniformly, not to perfectly match the real world. This means that more work needs to go into model calibration and parameterization before the results from this paper should hold any real policy relevance. At a basic level, this could involve a better thought out parameter values or could include the application of Bayesian methods to the model to fit the parameters.

Second, the model in the paper still makes use of representative agents. Although it has more heterogeneity than a RANK, our TANK only has two agents in it that are divided in a very specific way. A fully heterogeneous model could better analyze how representative agent assumptions effect our models than the TANK presented in this paper.

#### 6.2 Further Work

Extensions to the models presented in this paper should look at addressing these limitations, both through better parameter fitting and the addition of more layers of heterogeneity. This could include letting agents switch between being HtM and savers like in Bilbiie 2019 or abandoning HtM agents altogether and modeling productivity differences or idiosyncratic unemployment (McKay, Nakamura, and Steinsson 2015; McKay and Reis 2016b; Acharya and Dogra 2020).

# Appendices

## A Notes on the Model's Derivation

This follows Section 3 expanding on the methods used to derive the model. For clarity, equations that show up in Section 3 are numbered.

#### A.1 Firms, the Government, and the Central Bank

Final Goods Firm. Take the final goods firm production function

$$Y_{t} = \left(\int_{0}^{1} y_{j,t}^{\frac{\psi-1}{\psi}} \mathrm{d}j\right)^{\frac{\psi}{\psi-1}}$$
(A.1.1)

and profit maximization condition

$$\max_{y_{j,t}} P_t Y_t - \int_0^1 p_{j,t} y_{j,t} \mathrm{d}j.$$

Substituting the production function into the profit maximization condition gets

$$\max_{y_{j,t}} P_t \left( \int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}} \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} - \int_0^1 p_{j,t} y_{j,t} \mathrm{d}j$$

which has the FOC

$$\left(\frac{\psi-1}{\psi}\right)\left(\frac{\psi}{\psi-1}\right)y_{j,t}^{\frac{\psi-1}{\psi}-1}P_t\left(\int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}}\mathrm{d}j\right)^{\frac{\psi}{\psi-1}-1}=p_{j,t}.$$

Simplifying and rearranging this gets

$$y_{j,t}^{-\frac{1}{\psi}} = \frac{p_{j,t}}{P_t} \left( \int_0^1 y_{j,t}^{\frac{\psi-1}{\psi}} \mathrm{d}j \right)^{\frac{-1}{\psi-1}} = \frac{p_{j,t}}{P_t} Y_t^{-\frac{1}{\psi}}.$$

Raising this all to the  $-\psi$  gets

$$y_{j,t} = Y_t \left(\frac{P_t}{p_{j,t}}\right)^{\psi}.$$
(A.1.2)

Plugging Equation A.1.2 into Equation A.1.1 gets

$$\begin{split} Y_t &= \left( \int_0^1 \left( Y_t \left( \frac{P_t}{p_{j,t}} \right)^{\psi} \right)^{\frac{\psi-1}{\psi}} \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} \\ &= \left( \int_0^1 Y_t^{\frac{\psi-1}{\psi}} \left( \frac{P_t^{\psi-1}}{p_{j,t}^{\psi-1}} \right) \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} \\ &= \left( Y_t^{\frac{\psi-1}{\psi}} P_t^{\psi-1} \int_p^1 p_{j,t}^{1-\psi} \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} \\ &= Y_t P_t^{\psi} \left( \int_p^1 p_{j,t}^{1-\psi} \mathrm{d}j \right)^{\frac{\psi}{\psi-1}} . \end{split}$$

Dividing both sides by  $Y_t P_t^{\psi}$  gets

$$P_t^{-\psi} = \left(\int_p^1 p_{j,t}^{1-\psi} \mathrm{d}j\right)^{\frac{\psi}{\psi-1}}$$

which becomes the expression for the aggregate price level

$$P_t = \left(\int_p^1 p_{j,t}^{1-\psi} dj\right)^{\frac{1}{1-\psi}}.$$
 (A.1.3)

Intermediate Goods Firms. Taking the intermediate goods production function

$$y_{j,t} = Z_t n_{j,t}, \tag{A.1.4}$$

and demand for the jth intermediate good in Equation A.1.2 and combining them gets

$$Y_t\left(\frac{P_t}{p_{j,t}}\right)^{\psi} = Z_t n_{j,t}.$$

This means the cost minimization problem becomes

$$\min_{n_{j,t}} \quad W_t n_{j,t}$$
  
subject to  $Y_t \left(\frac{P_t}{p_{j,t}}\right)^{\psi} = Z_t n_{j,t}$ 

which has the Lagrangian

$$\mathcal{L} = W_t n_{j,t} + \lambda_{j,t} \left( Y_t \left( \frac{P_t}{p_{j,t}} \right)^{\psi} - Z_t n_{j,t} \right).$$

This gets the FOC

$$W_t = Z_t \lambda_{j,t}$$

where  $\lambda_{j,t}$  is the real marginal cost for the production of intermediate good j. Since the production function is linear and identical across goods,  $\lambda_{j,t} = \lambda_{j',t}$  for all intermediate goods j and j'. Therefore, we define  $\Lambda_t = \lambda_{j,t}$  and get

$$W_t = Z_t \Lambda_t. \tag{A.1.5}$$

Following Calvo 1983, intermediate goods firms update prices with probability  $1 - \theta$ . When updating prices, intermediate goods firms solve

$$\begin{aligned} \max_{P_t^*} \quad \mathbb{E}\sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \left( \frac{P_t^*}{P_s} y_{j,s} - W_s n_{j,s} \right) \\ \text{subject to} \quad W_s &= Z_s \Lambda_s \\ & Z_s n_{j,s} = y_{j,s} \\ & y_{j,s} = Y_s \left( \frac{P_s}{P_t^*} \right)^{\psi}. \end{aligned}$$

Plugging the first condition into the expression gets

$$\begin{split} \max_{P_t^*} \quad \mathbb{E}\sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \left( \frac{P_t^*}{P_s} y_{j,s} - Z_s \Lambda_s n_{j,s} \right) \\ \text{subject to} \quad Z_s n_{j,s} = y_{j,s} \\ \quad y_{j,s} = Y_s \left( \frac{P_s}{P_t^*} \right)^{\psi}. \end{split}$$

This becomes

$$\begin{split} \max_{P_t^*} \quad \mathbb{E}\sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \left( \frac{P_t^*}{P_s} y_{j,s} - \Lambda_s y_{j,s} \right) \\ \text{subject to} \quad y_{j,s} = Y_s \left( \frac{P_s}{P_t^*} \right)^{\psi}. \end{split}$$

which becomes

$$\max_{P_t^*} \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \left( Y_s \left( \frac{P_s}{P_t^*} \right)^{\psi-1} - \Lambda_s Y_s \left( \frac{P_s}{P_t^*} \right)^{\psi} \right).$$

This has the FOC

$$0 = \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \left( (\psi - 1) Y_s P_s^{\psi - 1} P_t^{*-\psi} + \psi \Lambda_s Y_s P_s^{\psi} P_t^{*-\psi - 1} \right)$$
  
$$= \mathbb{E} \sum_{s=t}^{\infty} (1 - \psi) \theta^{s-t} R_{t,s}^{-1} Y_s P_s^{\psi - 1} P_t^{*-\psi} + \mathbb{E} \sum_{s=t}^{\infty} \psi \theta^{s-t} R_{t,s}^{-1} \Lambda_s Y_s P_s^{\psi} P_t^{*-\psi - 1}$$
  
$$= (1 - \psi) P_t^{*-\psi} \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} Y_s P_s^{\psi - 1} + \psi P_t^{*-\psi - 1} \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \Lambda_s Y_s P_s^{\psi}.$$

Reorganizing this gets

$$(1-\psi)P_t^{*-\psi}\mathbb{E}\sum_{s=t}^{\infty}\theta^{s-t}R_{t,s}^{-1}Y_sP_s^{\psi-1} = \psi P_t^{*-\psi-1}\mathbb{E}\sum_{s=t}^{\infty}\theta^{s-t}R_{t,s}^{-1}\Lambda_sY_sP_s^{\psi}$$

which becomes

$$P_t^* = \frac{\psi \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \Lambda_s Y_s P_s^{\psi}}{(1-\psi) \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} Y_s P_s^{\psi-1}}.$$

Dividing both side by  $P_t$  gets

$$\frac{P_t^*}{P_t} = \frac{\psi \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \Lambda_s Y_s \left(\frac{P_s}{P_t}\right)^{\psi}}{(1-\psi) \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} Y_s \left(\frac{P_s}{P_t}\right)^{\psi-1}}.$$
(A.1.6)

To solve the problem, it'll also be helpful to define

$$P_{t}^{A} = \frac{\psi}{\psi - 1} \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} \Lambda_{s} Y_{s} \left(\frac{P_{s}}{P_{t}}\right)^{\psi} = \frac{\psi}{\psi - 1} \Lambda_{t} Y_{t} + \theta R_{t}^{-1} \mathbb{E} \pi_{t+1}^{\psi} P_{t+1}^{A}$$
$$P_{t}^{B} = \mathbb{E} \sum_{s=t}^{\infty} \theta^{s-t} R_{t,s}^{-1} Y_{s} \left(\frac{P_{s}}{P_{t}}\right)^{\psi - 1} = Y_{t} + \theta R_{t}^{-1} \mathbb{E} \pi_{t+1}^{\psi - 1} P_{t+1}^{B}$$

so that

$$\frac{P_t^*}{P_t} = \frac{P_t^A}{P_t^B}.$$

**Firms Aggregated.** Based on the Calvo rule, we know each period  $\theta$  of the firms keep their old price and  $(1 - \theta)$  change pries to  $P_t^*$ . Therefore,

$$P_{t} = \left(\int_{p}^{1} p_{j,t}^{1-\psi} dj\right)^{\frac{1}{1-\psi}}$$
$$= \left(\theta \int_{0}^{1} p_{j,t-1}^{1-\psi} dj + (1-\theta) \int_{0}^{1} P_{t}^{*1-\psi} dj\right)^{\frac{1}{1-\psi}}$$
$$= \left(\theta P_{t-1}^{1-\psi} + (1-\theta) P_{t}^{*1-\psi}\right)^{\frac{1}{1-\psi}}.$$

Dividing by  $P_t$ , this becomes

$$\begin{split} 1 &= \frac{\left(\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*1-\psi}\right)^{\frac{1}{1-\psi}}}{P_t} \\ &= \left(\frac{\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*1-\psi}}{P_t^{1-\psi}}\right)^{\frac{1}{1-\psi}} \\ &= \left(\theta \left(\frac{P_{t-1}}{P_t}\right)^{1-\psi} + (1-\theta) \left(\frac{P_t^*}{P_t}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}} \\ &= \left(\theta \pi_t^{\psi-1} + (1-\theta) \left(\frac{P_t^*}{P_t}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}} \end{split}$$

which, since the left hand side is 1, means

$$1 = \theta \pi_t^{\psi - 1} + (1 - \theta) \left(\frac{P_t^*}{P_t}\right)^{1 - \psi}$$
(A.1.7)

Combining Equation A.1.2 and Equation A.1.4 gets

$$Y_t \left(\frac{P_t}{p_{j,t}}\right)^{\psi} = Z_t n_{j,t}.$$

Integrating both sides gets

$$\int_0^1 Y_t \left(\frac{P_t}{p_{j,t}}\right)^{\psi} dj = \int_0^1 Z_t n_{j,t} dj$$
$$Y_t \int_0^1 \left(\frac{P_t}{p_{j,t}}\right)^{\psi} dj = Z_t N_t$$

which by defining

$$\begin{split} S_{t} &= \int_{0}^{1} \left(\frac{P_{t}}{p_{j,t}}\right)^{\psi} \mathrm{d}j \\ &= \theta \int_{0}^{1} \left(\frac{P_{t}}{p_{j,t-1}}\right)^{\psi} \mathrm{d}j + (1-\theta) \int_{0}^{1} \left(\frac{P_{t}}{P_{t}^{*}}\right)^{\psi} \mathrm{d}j \\ &= \theta \int_{0}^{1} \left(\frac{P_{t-1}P_{t}}{p_{j,t-1}P_{t-1}}\right)^{\psi} \mathrm{d}j + (1-\theta) \int_{0}^{1} \left(\frac{P_{t}}{P_{t}^{*}}\right)^{\psi} \mathrm{d}j \\ &= \theta \left(\frac{P_{t}}{P_{t-1}}\right)^{\psi} \int_{0}^{1} \left(\frac{P_{t-1}}{p_{j,t-1}}\right)^{\psi} + (1-\theta) \left(\frac{P_{t}}{P_{t}^{*}}\right)^{\psi} \\ S_{t} &= \theta S_{t-1} \left(\frac{P_{t}}{P_{t-1}}\right)^{\psi} + (1-\theta) \left(\frac{P_{t}}{P_{t}^{*}}\right)^{\psi} \end{split}$$

as the efficiency loss due to price dispersion following McKay and Reis 2016a gets

$$Y_t S_t = Z_t N_t.$$

In the paper, we present the equation as

$$Y_t = Z_t N_t \tag{A.1.8}$$

excluding  $S_t$  since it is equal to 1 in a first order approximation (McKay 2018). We can show this by loglinearizing the expression to get

$$S_{ss}\hat{S}_{t} = \theta S_{ss}\pi_{ss}^{\phi}(\hat{S}_{T} + \psi\hat{\pi}_{t}) + (1 - \theta)\left(\frac{P_{ss}}{P_{ss}^{*}}\right)^{\psi}(\psi\hat{P}_{t} - \psi\hat{P}_{t}^{*}).$$

Using  $\pi_{ss} = \frac{P_{ss}}{P_{ss}^*} = 1$  in the steady state gets

$$S_{ss} - \theta S_{ss} = 1 - \theta.$$

Additionally, rearranging the loglinear version of the nominal price condition

$$P_t^{1-\psi} = \theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*1-\psi}$$

gets

$$\hat{P}_t = \theta \hat{P}_{t-1} + (1-\theta)\hat{P}_t^*$$

and plugging in the equation for inflation gets

$$(1-\theta)(\hat{P}_t - \hat{P}_t^*) = -\theta(\hat{P}_t - \hat{P}_{t-1})$$
$$= -\theta\hat{\pi}_t$$

Plugging  $\pi_{ss} = \frac{P_{ss}}{P_{ss}^*} = S_{ss} = 1$  and  $(1 - \theta)(\hat{P}_t - \hat{P}_t^*) = -\theta\hat{\pi}_t$  into the expression gets

$$\hat{S}_t = \theta \hat{S}_T + \psi \theta \hat{\pi}_t - \psi \theta \hat{\pi}_t = \theta \hat{S}_T$$

which means  $\hat{S}_t = 0$  and  $S_t = S_{ss} = 1$ . Therefore, the expression for  $S_t$  and the  $S_t$  in the aggregate production function can be ignored.

Plugging the demand for  $y_{j,t}$  into the firm dividend expression

$$d_{j,t} = p_{j,t}y_{j,t} - P_t W_t n_{j,t}$$

gets

$$d_{j,t} = p_{j,t}Y_t \left(\frac{P_t}{p_{j,t}}\right)^{\psi} - P_t W_t n_{j,t}$$
$$= Y_t P_t^{\psi} p_{j,t}^{1-\psi} - P_t W_t n_{j,t}.$$

Integrating both sides means

$$\int_0^1 d_{j,t} dj = \int_0^1 Y_t P_t^{\psi} p_{j,t}^{1-\psi} - P_t W_t n_{j,t} dj$$
$$P_t D_t = Y_t P_t^{\psi} \int_0^1 p_{j,t}^{1-\psi} dj - P_t W_t \int_0^1 n_{j,t} dj.$$

Plugging in the aggregate price and labor equations into this gets

$$D_t = Y_t - W_t N_t. \tag{A.1.9}$$

## A.2 RANK Model

**Representative Household.** The representative household solves

$$\max_{\{C_t, L_t, B_t\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$
  
subject to  $C_t + B_t = R_{t-1}B_{t-1} + W_t L_t + D_t - T_t.$ 

This has the Lagrangian

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^t \left( U(C_t, L_t) + \lambda_{1,t} (R_{t-1}B_{t-1} + W_t L_t + D_t - T_t - C_t - B_t) \right)$$

which has the FOCs

$$\lambda_{1,t} = U_c(C_t, L_t) \tag{C_t}$$

$$\lambda_{1,t} = \frac{-U_L(C_t, L_t)}{W_t} \tag{L}_t$$

$$\lambda_{1,t} = \beta R_t \mathbb{E} \lambda_{1,t+1}. \tag{B_t}$$

Combining these at  $\lambda_{1,t}$  gets the intratemporal condition

$$W_t = \frac{-U_L(C_t, L_t)}{U_c(C_t, L_t)}$$

and Euler equation

$$U_C(C_t, L_t) = \beta R_t \mathbb{E} U_C(C_{t+1}, L_{t+1})$$
$$1 = \beta R_t \mathbb{E} \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}.$$

Since the utility function

$$U(C_t, L_t) = \frac{C_t^{1-\eta}}{1-\eta} - \phi \frac{L_t^{1+\chi}}{1-\chi}$$
(A.2.1)

has the FOCs

$$U_C(C_t, L_t) = C_t^{-\eta} \tag{C_t}$$

$$U_L(C_t, L_t) = -\phi L_t^{\chi}, \qquad (L_t)$$

these become the Euler Equation

$$1 = \beta R_t \mathbb{E} \frac{C_t^{\eta}}{C_{t+1}^{\eta}} \tag{A.2.2}$$

and intratemporal condition

$$\phi L_t^{\chi} C_t^{\eta} = W_t. \tag{A.2.3}$$

## A.3 TANK Model

Hand-to-Mouth Households. Each period, HtM households solve

$$\max_{C_t^H, L_t^H} \quad U(C_t^H, L_t^H)$$
  
subject to  $C_t^H = W_t L_t^H + D_t - T_t.$ 

This gets the Lagrangian

$$\mathcal{L} = U(C_t^H, L_t^H) - \lambda_{2,t}(W_t L_t^H + D_t - T_t - C_t^H)$$

which has the FOCs

$$\lambda_{2,t} = U_C(C_t^H, L_t^H) \tag{C}_t^H$$

$$\lambda_{2,t} = \frac{-U_L(C_t^H, L_t^H)}{W_t} \tag{L}_t^H$$

Combining these at  $\lambda_{2,t}$  gets the intratemporal condition

$$W_t = \frac{-U_L(C_t^H, L_t^H)}{U_C(C_t^H, L_t^H)}$$

Using the utility function

$$\frac{C_t^{H1-\eta}}{1-\eta} - \phi \frac{L_t^{H1+\chi}}{1+\chi}$$
(A.3.1)

with FOCs

$$U_C(C_t^H, L_t^H) = C_t^{H-\eta}$$
$$U_L(C_t^H, L_t^H) = -\phi L_t^{H\chi}$$

gets the intratemporal condition

$$W_t = \phi L_t^{H\chi} C_t^{H\eta}. \tag{A.3.2}$$

Saver Households. Each period, the saver household solves

$$\begin{split} \max_{C_t^S, L_t^S, B_t} \quad & \mathbb{E}\sum_{t=0}^\infty \beta^t U(C_t^S, L_t^S) \\ \text{subject to} \quad & C_t^S + B_t^S = R_{t-1}B_{t-1}^S + W_t L_t^S + D_t - T_t. \end{split}$$

This gets the Lagrangian

$$\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^{t} \left( U(C_{t}^{S}, L_{t}^{S}) + \lambda_{3,t} \left( R_{t-1}B_{t-1}^{S} + W_{t}L_{t}^{S} + D_{t} - T_{t} - C_{t}^{S} - B_{t}^{S} \right) \right)$$

which has the FOCs

$$\lambda_{3,t} = U_c(C_t^S, L_t^S) \tag{C}_t^S$$

$$\lambda_{3,t} = \frac{-U_L(C_t^S, L_t^S)}{W_t} \tag{L}_t^S$$

$$\lambda_{3,t} = \beta R_t \mathbb{E}\lambda_{1,t+1}.\tag{B}^S_t$$

Combining at  $\lambda_{3,t}$  gets the intratemporal condition

$$W_t = \frac{-U_L(C_t^S, L_t^S)}{U_C(C_t^S, L_t^S)}$$

and Euler Equation

$$U_C(C_t, L_t) = \beta R_t \mathbb{E} U_C(C_{t+1}, L_{t+1})$$
$$1 = \beta R_t \mathbb{E} \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}$$

Plugging in the utility function

$$\frac{C_t^{S1-\eta}}{1-\eta} - \phi \frac{L_t^{S1+\chi}}{1+\chi}$$
(A.3.3)

with FOCs

$$U_C(C_t^S, L_t^S) = C_t^{S-\eta}$$
$$U_L(C_t^S, L_t^S) = -\phi L_t^{S\chi}$$

gets the intratemporal condition

$$W_t = \phi L_t^{S\chi} C_t^{S\eta} \tag{A.3.4}$$

and Euler Equation

$$1 = \beta R_t \mathbb{E} \frac{C_t^{S\eta}}{C_{t+1}^{S\eta}}.$$
(A.3.5)

## **B** Solution Methods

## **B.1 RANK Solution**

**Solution System.** Before we solve the system from Section 3.2, we'll make a couple modifications to make it tractable and simplify it. First, we'll replace  $\frac{P_t^*}{P_t}$  with  $\frac{P_t^A}{P_t^B}$  and add the definitions to the system. We'll also eliminate the labor market clearing conditions and plug  $N_t$  in for  $L_t$  everywhere. Finally, we'll plug in the dividend expression and government budget into the household budget to

 $\operatorname{get}$ 

$$C_t + B_t = R_{t-1}B_{t-1} + W_t N_t + Y_t - W_t N_t + B_t - R_{t-1}B_{t-1} - G_t$$

which then substituting the goods market clearing condition means

$$C_t + B_t = R_{t-1}B_{t-1} + W_tN_t + C_t + G_t - W_tN_t + B_t - R_{t-1}B_{t-1} - G_t$$

which simplifies to 0 = 0. Therefore, we exclude the government and household budgets and eliminate  $T_t$  and  $B_t$  from the system.

Therefore, our solution is the set  $\{W_t, Y_t, C_t, N_t, G_t, \Lambda_t, \pi_t, P_t^A, P_t^B, I_t, R_t\}_{t=0}^{\infty}$  that satisfies

$$\begin{split} W_t &= \phi N_t^{\chi} C_t^{\eta} \\ 1 &= \beta R_t \mathbb{E} \frac{C_t^{\eta}}{C_{t+1}^{\eta}} \\ \Lambda_t &= Z_t W_t \\ 1 &= \theta \pi_t^{\psi - 1} + (1 - \theta) \left(\frac{P_t^A}{P_t^B}\right)^{1 - \psi} \\ P_t^A &= \frac{\psi}{\psi - 1} \Lambda_t Y_t + \theta R_t^{-1} \mathbb{E} \pi_{t+1}^{\psi} P_{t+1}^A \\ P_t^B &= Y_t + \theta R_t^{-1} \mathbb{E} \pi_{t+1}^{\psi - 1} P_{t+1}^B \\ Y_t &= Z_t N_t \\ G_t &= g_t Y_t \\ I_t &= \overline{R} \pi_t^{\omega} \xi_t \\ R_t &= \mathbb{E} \frac{I_t}{\pi_{t+1}} \\ Y_t &= C_t + G_t \end{split}$$

subject to the exogenous processes

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t}$$
$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_{\xi,t}$$
$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\overline{g} + \varepsilon_{g,t}.$$

Steady States. In the steady state, we know

$$\begin{split} W_{ss} &= \phi N_{ss}^{\chi} C_{ss}^{\eta} \\ 1 &= \beta R_{ss} \frac{Y_{ss}^{\eta}}{Y_{ss}^{\eta}} \\ \Lambda_{ss} &= Z_{ss} W_{ss} \\ 1 &= \theta \pi_{ss}^{\psi-1} + (1-\theta) \left(\frac{P_{ss}^{A}}{P_{ss}^{B}}\right)^{1-\psi} \\ P_{ss}^{A} &= \frac{\psi}{\psi-1} \Lambda_{ss} Y_{ss} + \theta R_{ss}^{-1} \pi_{ss}^{\psi} P_{ss}^{A} \\ P_{ss}^{B} &= Y_{ss} + \theta R_{ss}^{-1} \mathbb{E} \pi_{ss}^{\psi-1} P_{ss}^{B} \\ Y_{ss} &= Z_{ss} N_{ss} \\ G_{ss} &= g_{ss} Y_{ss} \\ I_{ss} &= \overline{R} \pi_{ss}^{\omega} \xi_{ss} \\ R_{ss} &= \frac{I_{ss}}{\pi_{ss}} \\ Y_{ss} &= C_{ss} + G_{ss} \end{split}$$

Since we're assuming a zero-inflation steady state, we know  $\pi_{ss} = 1$ . Furthermore, the steady state for the exogenous shocks  $Z_{ss} = \xi_{ss} = 1$  and  $g_{ss} = \overline{g}$ .

The second equation means

$$R_{ss} = \frac{1}{\beta}.$$

The tenth then means

$$I_{ss} = R_{ss} = \frac{1}{\beta}.$$

The fourth equation gets

$$1 - \theta = (1 - \theta) \left(\frac{P_{ss}^A}{P_{ss}^B}\right)^{1 - \psi}$$

which becomes

$$1 = \left(\frac{P^A_{ss}}{P^B_{ss}}\right)^{1-\psi}$$

which means

$$P_{ss}^A = P_{ss}^B$$

The fifth equation becomes

$$(1 - \beta\theta)P_{ss}^A = \frac{\psi}{\psi - 1}\Lambda_{ss}Y_{ss}$$

and, using  $P_{ss}^A = P_{ss}^B$ , the sixth equation becomes

$$(1 - \beta\theta)P_{ss}^A = Y_{ss}.$$

Together, these mean

$$Y_{ss} = \frac{\psi}{\psi - 1} \Lambda_{ss} Y_{ss}$$

which gets

$$\Lambda_{ss} = \frac{\psi - 1}{\psi}.$$

By the third equation, we know

$$W_{ss} = \Lambda_{ss} = \frac{\psi - 1}{\psi}.$$

The seventh, eighth, and eleventh equations get

$$N_{ss} = Y_{ss}$$
$$G_{ss} = \overline{g}Y_{ss}$$
$$C_{ss} = (1 - \overline{g})Y_{ss}$$

which substituted into the first equation means

$$\frac{\psi - 1}{\psi} = \phi Y_{ss}^{\chi} \left( (1 - \overline{g}) Y_{ss} \right)^{\eta}$$
$$= \phi (1 - \overline{g})^{\eta} Y_{ss}^{\chi + \eta}$$

which means

$$Y_{ss}^{\chi+\eta} = \frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}.$$

Therefore,

$$Y_{ss} = \left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}.$$

This combined with our expressions for  $N_{ss}$ ,  $G_{ss}$ ,  $C_{ss}$ ,  $P_{ss}^A$ , and  $P_{ss}^B$  in terms of  $Y_{ss}$  get us the

steady state of the RANK,

$$g_{ss} = \overline{g}$$

$$\pi_{ss} = 1$$

$$Z_{ss} = 1$$

$$Z_{ss} = 1$$

$$I_{ss} = \frac{1}{\beta}$$

$$R_{ss} = \frac{1}{\beta}$$

$$R_{ss} = \frac{\psi - 1}{\psi}$$

$$W_{ss} = \frac{\psi - 1}{\psi}$$

$$W_{ss} = \left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}$$

$$R_{ss} = \left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}$$

$$G_{ss} = \overline{g} \left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}$$

$$C_{ss} = (1 - \overline{g}) \left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}$$

$$P_{ss}^{A} = \frac{\left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}}{1 - \beta\theta}$$

$$P_{ss}^{B} = \frac{\left(\frac{\psi - 1}{(1 - \overline{g})^{\eta}\psi\phi}\right)^{\frac{1}{\chi + \eta}}}{1 - \beta\theta}$$

To solve the model, we use the Jacobian of the system evaluated at this steady state and Linear Time Iteration from Rendahl 2017.

## B.2 TANK Solution

**Solution System.** Similar to the RANK, before we solve the system from Section 3.3, we'll make a couple modifications to make it tractable and simpler. First, we'll add  $P_t^A$  and  $P_t^B$  and

their definitions into the system and replace  $\frac{P_t^*}{P_t}$  with  $\frac{P_t^A}{P_t^B}$ . Next, we'll eliminate  $B_t$  and  $L_t$  from the system and plug in the expressions  $B_t = (1-\mu)B_t^S$  and  $L_t = \mu L_t^H + (1-\mu)L_t^S$  for them everywhere. Finally, we'll eliminate  $D_t$  from the system and plug its definition in everywhere it shows up. This means the HtM budget constraint becomes

$$C_t^H = W_t (L_t^H - N_t) + Y_t - T_t$$

and the saver budget becomes

$$C_t^S + B_t^S = R_{t-1}B_{t-1} + W_t(L_t^S - N_t) + Y_t - T_t.$$

Thus, our solution is the set  $\{W_t, Y_t, C_t, C_t^H, C_t^S, N_t, L_t^H, L_t^S, B_t^S, G_t, T_t, \Lambda_t, \pi_t, P_t^A, P_t^B, I_t, R_t\}_{t=0}^{\infty}$ 

that satisfies

$$\begin{split} C_t^H &= W_t(L_t^H - N_t) + Y_t - T_t \\ W_t &= \phi L_t^{H\chi} C_t^{H\eta} \\ C_t^S + B_t^S &= R_{t-1} B_{t-1}^S + W_t(L_t^S - N_t) + Y_t - T_t \\ W_t &= \phi L_t^{S\chi} C_t^{S\eta} \\ 1 &= \beta R_t \mathbb{E} \frac{C_t^{S\eta}}{C_{t+1}^{S\eta}} \\ \Lambda_t &= Z_t W_t \\ 1 &= \theta \pi_t^{\psi - 1} + (1 - \theta) \left(\frac{P_t^A}{P_t^B}\right)^{1 - \psi} \\ P_t^A &= \frac{\psi}{\psi - 1} \Lambda_t Y_t + \theta R_t^{-1} \mathbb{E} \pi_{t+1}^{\psi} P_{t+1}^A \\ P_t^B &= Y_t + \theta R_t^{-1} \mathbb{E} \pi_{t+1}^{\psi - 1} P_{t+1}^B \\ Y_t &= Z_t N_t \\ G_t &= R_t \mathcal{E}_t \\ G_t &= g_t Y_t \\ I_t &= \overline{R} \pi_t^{\omega} \xi_t \\ R_t &= \mathbb{E} \frac{I_t}{\pi_{t+1}} \\ N_t &= \mu L_t^H + (1 - \mu) L_t^S \\ C_t &= \mu C_t^H + (1 - \mu) C_t^S \\ Y_t &= C_t + G_t \end{split}$$

subject to the exogenous processes

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t}$$
$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_{\xi,t}$$
$$g_t = \rho_g g_{t-1} + (1 - \rho_g)\overline{g} + \varepsilon_{g,t}.$$

Steady States. In the steady state, we know

$$\begin{split} C^H_{ss} &= W_{ss}(L^H_{ss} - N_{ss}) + Y_{ss} - T_{ss} \\ (1 - \tau_{ss})W_{ss} &= \phi L^{H\chi}_{ss} C^{H\eta}_{ss} \\ C^S_{ss} + B^S_{ss} &= R_{ss} B^S_{ss} + W_{ss}(L^S_{ss} - N_{ss}) + Y_{ss} - T_{ss} \\ (1 - \tau_{ss})W_{ss} &= \phi L^{S\chi}_t C^{S\eta}_{ss} \\ 1 &= \beta R_{ss} \frac{C^{S\eta}_{ss}}{C^{S\eta}_{ss}} \\ \Lambda_{ss} &= Z_{ss} W_{ss} \\ 1 &= \theta \pi^{\psi-1}_{ss} + (1 - \theta) \left(\frac{P^A_{ss}}{P^B_{ss}}\right)^{1-\psi} \\ P^A_{ss} &= \frac{\psi}{\psi - 1} \Lambda_{ss} Y_{ss} + \theta R^{-1}_{ss} \pi^{\psi}_{ss} P^A_{ss} \\ P^B_{ss} &= Y_{ss} + \theta R^{-1}_{ss} \pi^{\psi-1}_{ss} P^B_{ss} \\ Y_{ss} &= Z_{ss} N_{ss} \\ G_{ss} + R_{ss}(1 - \mu) B^S_{ss} &= T_{ss} + (1 - \mu) B^S_{ss} \\ G_{ss} &= g_{ss} Y_{ss} \\ I_{ss} &= \overline{R} \pi^{\omega}_{ss} \xi_{ss} \\ R_{ss} &= \frac{I_{ss}}{\pi_{ss}} \\ N_{ss} &= \mu L^H_{ss} + (1 - \mu) L^S_{ss} \\ C_{ss} &= \mu C^H_{ss} + (1 - \mu) C^S_{ss} \\ Y_{ss} &= C_{ss} + G_{ss} \\ \end{split}$$

Since we're loglinearizing around the zero-inflation steady state, we know  $\pi_{ss} = 1$ . Furthermore, the steady state for the exogenous shocks  $Z_{ss} = \xi_{ss} = 1$  and  $g_{ss} = \overline{g}$ . We'll also need to define  $B_{ss} = B$ , which means  $B_{ss}^S = \frac{B}{1-\mu}$ .

The fifth equation means

$$R_{ss} = \frac{1}{\beta}.$$

It follows from the fourteenth equation that

$$I_{ss} = R_{ss} = \frac{1}{\beta}.$$

The seventh equation gets

$$1 - \theta = (1 - \theta) \left(\frac{P_{ss}^A}{P_{ss}^B}\right)^{1 - \psi}$$

which becomes

$$1 = \left(\frac{P^A_{ss}}{P^B_{ss}}\right)^{1-\psi}$$

which means

$$P_{ss}^A = P_{ss}^B.$$

The eighth equation becomes

$$(1 - \beta\theta)P_{ss}^A = \frac{\psi}{\psi - 1}\Lambda_{ss}Y_{ss}$$

and, using  $P^A_{ss}=P^B_{ss},$  the ninth equation becomes

$$(1 - \beta\theta)P_{ss}^A = Y_{ss}.$$

Together, these mean

$$Y_{ss} = \frac{\psi}{\psi - 1} \Lambda_{ss} Y_{ss}$$

which gets

$$\Lambda_{ss} = \frac{\psi - 1}{\psi}.$$

By the sixth equation, we know

$$W_{ss} = \Lambda_{ss} = \frac{\psi - 1}{\psi}.$$

We have

$$N_{ss} = Y_{ss}$$

from the tenth equation,

$$G_{ss} = \overline{g}Y_{ss},$$

from the twelfth,

$$C_{ss} = (1 - \overline{g})Y_{ss},$$

from the seventeenth, and

$$T_{ss} = \frac{1-\beta}{\beta}B + \overline{g}Y_{ss}$$

from the eleventh.

Using what we know, the second equation means

$$L_{ss}^{H} = \left(\frac{\psi - 1}{\psi\phi}\right)^{\frac{1}{\chi}} C_{t}^{H - \frac{\eta}{\chi}}$$

and the fourth means

$$L_{ss}^{S} = \left(\frac{\psi - 1}{\psi\phi}\right)^{\frac{1}{\chi}} C_{t}^{S - \frac{\eta}{\chi}}.$$

Using these, the aggregators from the fifteenth and sixteenth equations, and the first equation, we get the system

$$\begin{split} L^{H}_{ss} &= \left(\frac{\psi-1}{\psi\phi}\right)^{\frac{1}{\chi}} C^{H-\frac{\eta}{\chi}}_{t} \\ L^{S}_{ss} &= \left(\frac{\psi-1}{\psi\phi}\right)^{\frac{1}{\chi}} C^{S-\frac{\eta}{\chi}}_{t} \\ \mu L^{H}_{ss} + (1-\mu) L^{S}_{ss} &= \frac{\mu C^{H}_{ss} + (1-\mu) C^{S}_{ss}}{1-\overline{g}} \\ C^{H}_{ss} - C^{S}_{ss} + \frac{1-\beta}{(1-\mu)\beta} B &= \frac{\psi-1}{\psi} (L^{H}_{ss} - L^{S}_{ss}) \end{split}$$

which we solve using Newton's Method.

Therefore, the steady state of the RANK is

$$\begin{split} g_{ss} &= \overline{g} \\ B_{ss} &= B \\ \pi_{ss} &= 1 \\ Z_{ss} &= 1 \\ Z_{ss} &= 1 \\ \zeta_{ss} &= \frac{1}{\beta} \\ R_{ss} &= \frac{1}{\beta} \\ R_{ss} &= \frac{\psi - 1}{\psi} \\ W_{ss} &= \frac{\psi - 1}{\psi} \\ W_{ss} &= \frac{\psi - 1}{\psi} \\ C_{ss} &= \mu C_{ss}^{H} + (1 - \mu) C_{ss}^{S} \\ Y_{ss} &= \frac{\mu C_{ss}^{H} + (1 - \mu) C_{ss}^{S}}{1 - \overline{g}} \\ N_{ss} &= \mu L_{ss}^{H} + (1 - \mu) L_{ss}^{S} \\ G_{ss} &= \overline{g} \left( \frac{\mu C_{ss}^{H} + (1 - \mu) C_{ss}^{S}}{1 - \overline{g}} \right) \\ P_{ss}^{A} &= \frac{\mu C_{ss}^{H} + (1 - \mu) C_{ss}^{S}}{(1 - \overline{g})(1 - \beta \theta)} \\ P_{ss}^{B} &= \frac{\mu C_{ss}^{H} + (1 - \mu) C_{ss}^{S}}{(1 - \overline{g})(1 - \beta \theta)} \\ T_{ss} &= \frac{1 - \beta}{\beta} B + \overline{g} \left( \frac{\mu C_{ss}^{H} + (1 - \mu) C_{ss}^{S}}{1 - \overline{g}} \right) \end{split}$$

where  $C_{ss}^{H}, C_{ss}^{S}, L_{ss}^{H}$ , and  $L_{ss}^{S}$  come from Newton's method.

# C Calibration

Based on the steady state versions of the Euler equation and the Taylor Rule, we get

$$I_{ss} = \frac{1}{\beta} = \overline{R}.$$

Therefore, our calibrated values are chosen to create this equivalence.

# D Results

# D.1 Simulation



Figure D.1: Stochastic Paths for Our Simulation

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